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# Optimal Three Burn Orbit Transfer


Engineering Science Operations
The Aerospace Corporation
El Segundo, Calif. 90245




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A. G. Fernandez, 1st Lt, USAF
Project Engineer

Joseph Gassmann, Major, USAF

FOR THE COMMANDER

Floyd R. Stuart, Colonel, USAF
Deputy for Advanced Space Programs

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This report presents an optimal three burn solution to a class of orbit transfer problems requiring large changes in orbital inclination. A non-linear programming algorithm was used in conjunction with a simplified trajectory simulation, which used Keplerian orbit transfers and impulsive velocity increments. A number of parametric results were obtained using the simplified simulation. The validity of the simplified simulation was

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established for specific vehicle configurations with finite burns and oblate earth effects included. The effect of multiple stage rockets has also been treated.


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# PREFACE

The author gratefully acknowledges the continuing assistance and insight supplied by A. D. Hemenover in the development and preparation of these results.

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## 1. INTRODUCTION

Plans for using the space shuttle orbiter vehicle to launch a number of payloads have been under recent investigation. It is proposed that the shuttle be launched from the Eastern Test Range inserting the orbiter into a park orbit.

An interim upper stage (IUS) which is to be deployed from the orbiter and used to achieve the desired final orbit has been of interest to the Air Force as well as a number of private contractors.

The park orbit was defined by a number of considerations to be a circular orbit with altitude of 160 n mi and inclination $37.4^{\circ}$. It was desired that the final orbit have an inclination of $63.4^{\circ}$, an argument of perigee of $270^{\circ}$, and a period of twelve hours. The specific twelve hour orbit of interest has an apogee altitude of 21450 n mi and a perigee altitude of 350 n mi. It became apparent that most of the proposed IUS configurations could not perform the orbit transfer from park orbit to final orbit using two burns, and still yield some desired payload values. Consequently a study was initiated to investigate the feasibility of an optimal three burn orbit transfer.

This report presents the results of the optimal three burn orbit transfer investigation. Initially the definition of a simplified orbit transfer simulation, using Keplerian orbit transfers and impulsive velocity increments, is given. After discussing the use of a nonlinear programming algorithm to solve this simplified problem, the results for a family of final orbits with different inclinations are presented. The accuracy of the simplified simulation has been verified by comparison with a detailed trajectory simulation.

## 2. THE ORBIT TRANSFER PROBLEM

### a) Three Burn Impulsive Velocity Orbit Transfer

Let us begin with the definition of a simplified orbit transfer simulation.

1) Specify the initial state: the position vector $y(t_o)$, the velocity vector $\dot{y}(t_o)$, and the initial time $t_o$. We assume that the state is specified using an earth-centered inertial (ECI) coordinate system, and is a point on the 160 n mi. circular park orbit with inclination $37.4^o$. Furthermore we shall assume that a spherical earth model describes the geopotential function.

2) Coast in the orbit until time = $t_1 = t_o + \Delta t_1$, where $\Delta t_1$ is the length of the first coast in seconds. Essentially the coast is simulated by integrating the equations of motion, $\ddot{y} = -\mu \| y \|^{-3} y$, from $t_o$ to $t_1$. The result of the integration is the state vector $y(t_1)$, $\dot{y}(t_1)$ and because of the spherical geometry assumption this integration can be performed quite rapidly. The complete algorithm for two-body motion in space used in the simulation is described in Escobal [1].

3) Simulate the first burn, by adding to the inertial velocity vector $\dot{y}(t_1)$ the velocity increment defined by $(\Delta V_1, \Delta\theta_1, \Delta\psi_1)$. The pitch increment $\Delta\theta_1$ and the yaw increment $\Delta\psi_1$ are defined with respect to the local inertial velocity as shown in Figure 1, and are applied as a yaw-pitch-roll sequence.

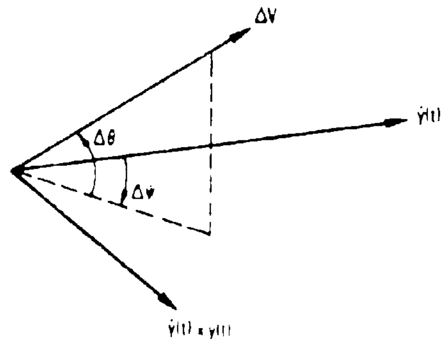


**Figure 1. Definition of Yaw Increment, $\Delta\psi$, and Pitch Increment, $\Delta\theta$**

4) Coast in the first transfer orbit as defined by the new state vector until time = $t_2 = t_1 + \Delta t_2$.

5) Simulate the second burn, i.e., add to the inertial velocity vector the increment defined by $(\Delta V_2, \Delta\theta_2, \Delta\psi_2)$.

6) Coast in the second transfer orbit until time = $t_3 = t_2 + \Delta t_2$

7) Simulate the third burn - add the increment defined by $(\Delta V_3, \Delta\theta_3, \Delta\psi_3)$.

8) End of Simulation - Final Orbit.

It is important to note that the coast times $\Delta t_i$, and the velocity increments $(\Delta V_i, \Delta\theta_i, \Delta\psi_i)$ $i = 1, 2, 3$, in conjunction with the initial conditions completely determine the final orbit. The entire sequence of events was simulated using the Generalized Trajectory Simulation (GTS) [2].

b) The Optimization Problem

The sequence of steps required to compute the final orbit have been outlined. Explicitly one must choose the twelve variables

$$\vec{x} = (\Delta t_1, \Delta t_2, \Delta t_3, \Delta V_1, \Delta V_2, \Delta V_3, \Delta\theta_1, \Delta\psi_1, \Delta\theta_2, \Delta\psi_2, \Delta\theta_3, \Delta\psi_3)$$

such that the objective function

$$f(\vec{x}) = \Delta V_1 + \Delta V_2 + \Delta V_3 \tag{1}$$

is minimized and the constraints

$$c_1(\vec{x}) = h_a - 21,450 = 0. \tag{2}$$

$$c_2(\vec{x}) = h_p - 350 = 0. \tag{3}$$

$$c_3(\vec{x}) = \omega_p - 270 = 0. \tag{4}$$

$$c_4(\vec{x}) = i - 63.4 = 0. \tag{5}$$

$$c_5(\vec{x}) = 75,000 - h_a(t_1) \geq 0 \tag{6}$$

$$c_6(\vec{x}) = 75,000 - h_a(t_2) \geq 0 \tag{7}$$

are satisfied, where $h_a$ is the apogee altitude (n mi), $h_p$ the perigee altitude (n mi), $\omega_p$ the argument of perigee (deg), and $i$ the inclination (deg).

The first two constraints on apogee and perigee altitude $h_a$ and $h_p$ are representative of the family of twelve hour period orbits required. The third constraint requires that the argument of perigee, $\omega_p$ be 270° and the fourth constraint defines the final orbit inclination. The last two constraints establish limits on the apogee altitudes of the first and second transfer orbits respectively, and are included to prevent escape orbits during the transfer. The 75000 n mi limit is somewhat arbitrary and its significance will be discussed later.

## 3. THE NONLINEAR PROGRAMMING PROBLEM

The previous section outlined an example of a nonlinear programming problem. Stated concisely the general problem is to determine the n-vector $x = (x_1, x_2 \ldots x_n)$ that minimizes (maximizes) the objective function

$$f(x) = f(x_1, x_2, \ldots, x_n) \tag{8}$$

subject to the equality constraints

$$c_i(x) = 0 \qquad i = 1 \ldots k \tag{9}$$

and the inequality constraints

$$c_i(x) \geq 0 \qquad i = (k+1), \ldots, m. \tag{10}$$

Define the Lagrangian

$$L(x, \lambda) = f(x) + c^T(x)\, \lambda \tag{11}$$

where $\lambda$ is the m-vector of Lagrange multipliers. The Kuhn-Tucker necessary conditions require that at the optimum point $(x^*, \lambda^*)$

$$\nabla L(x^*, \lambda^*) = g(x^*) + G(x^*)\, \lambda^* = 0 \tag{12}$$

where g(x) is the gradient vector of f(x), and the $n \times m$ Jacobian matrix is defined by

$$G(x) = [\nabla c_1, \ldots, \nabla c_m] = \begin{bmatrix} \frac{\partial c_1}{\partial x_1} & \cdots & \frac{\partial c_m}{\partial x_1} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \frac{\partial c_1}{\partial x_n} & \cdots & \frac{\partial c_m}{\partial x_n} \end{bmatrix} \qquad (13)$$

Also at $(x^*, \lambda^*)$

$$\lambda_i^* \, c_i(x^*) = 0 \qquad i = 1, \ldots, m \qquad (14)$$

where $\lambda_i^* \leq 0 \qquad i = (k+1), \ldots m \qquad (15)$

In order to distinguish the set of constraints satisfied as equality constraints at the solution introduce the basic set of constraints

$$B^* = \left\{ i \mid c_i(x^*) = 0; \qquad i = 1, \ldots, m \right\} \qquad (16)$$

Assume that the gradients of the constraints in $B^*$ are linearly independent at $x^*$.

The solution of the nonlinear programming problem has received a great deal of attention in recent years and a number of algorithms have appeared in the literature. The accelerated multiplier algorithm[3] was used to solve the stated orbit transfer problem. Essentially the algorithm consists of a number of cycles through the following steps:

Step 1. Find a point where $\nabla L(x, \lambda) = 0$, for fixed $\lambda$, by finding the unconstrained minimum of an augmented performance index. A rank one recursive algorithm is used for the unconstrained optimization process.

Step 2. Estimate the Lagrange multipliers $\lambda$, by minimizing the error in the Kuhn-Tucker conditions. Using the multipliers, make an estimate B for the basic set of constraints $B^*$.

Step 3. Extrapolate to find a point where $c_i(x) = 0$, assuming the objective function is quadratic and the constraints are linear.

Step 4. Reestimate the multipliers $\lambda$, and the basis B.

Step 5. Test for Convergence.

Typically two or three cycles are required to obtain the desired accuracy. For a more detailed description of the nonlinear programming algorithm, the reader is referred to Ref. 3-5.

## 4. ORBIT TRANSFER RESULTS

The nonlinear programming algorithm given above was applied to the stated orbit transfer problem and Figure 2 summarizes the results. After coasting in the park orbit the first burn added 7820 fps to the inertial velocity vector with very little pitch offset ($-1.36^{\circ}$) and yaw offset ($5.99^{\circ}$). The resulting transfer orbit had an apogee altitude of 17,617 n mi, while keeping the perigee altitude and inclination essentially unchanged. After coasting to an altitude, h, of 10,838 n mi, where the argument of latitude, u, was $37.7^{\circ}$, the second burn was performed. An impulsive velocity of 4054 fps was added with a yaw left of $74.7^{\circ}$ and a positive pitch angle of $7.21^{\circ}$. Because of this out of plane maneuver, most of the required plane change was accomplished, the second transfer orbit having an inclination of $58.3^{\circ}$. Notice also that the second transfer orbit has an apogee altitude of 23,386 n mi and a perigee altitude of 1,314 n mi and hence has more energy than the desired final orbit. The third and final burn occurred when the argument of latitude was

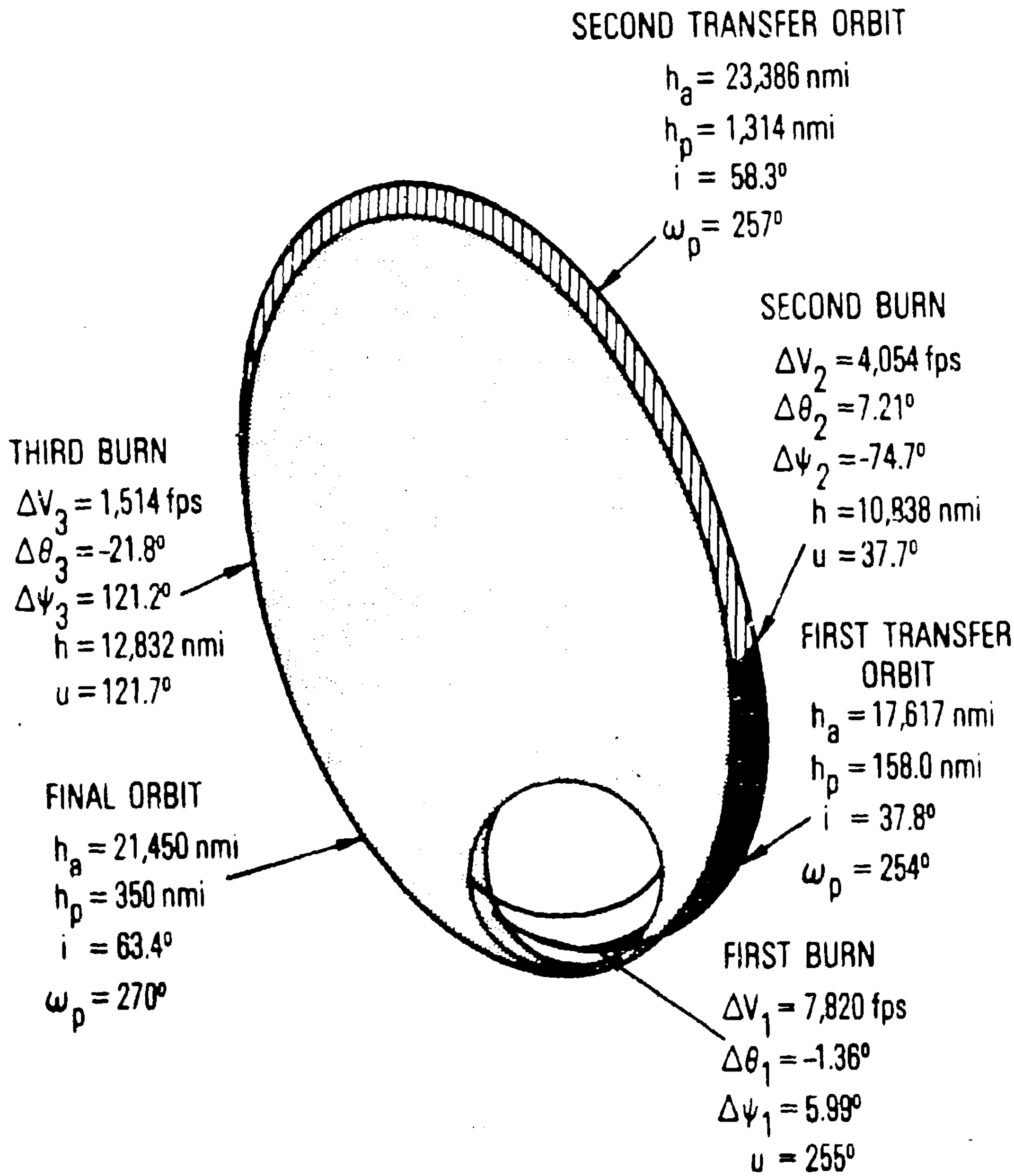


Figure 2. Optimal Three Burn Transfer, i = 63.4°

$121.7^{\circ}$ at an altitude of 12,832 n mi, and was essentially a retro burn which added 1,514 fps velocity with a yaw attitude of $121.2^{\circ}$ and pitch down of $21.8^{\circ}$. The retro maneuver corrected the argument of perigee error in the second transfer orbit while also reducing the energy of the final orbit.

After obtaining results for the final orbit with inclination of $63.4^{\circ}$, a number of other optimal solutions were obtained for larger inclinations. In particular, Figure 3 presents the optimal three burn orbit with an inclination of $116.6^{\circ}$, which required a plane change of $79.2^{\circ}$ from the park orbit. All other constraints in the original problem statement remain unchanged. The first burn added 9686 fps velocity approximately along the inertial velocity vector ($\Delta\theta_1 = -.113^{\circ}$, $\Delta\psi_1 = 1.080$). As in the first case, the resulting transfer orbit remained nearly unchanged in all orbital elements except the apogee altitude which was increased to 72,364 n mi. After coasting to an altitude of 65,041 n mi, an additional 3,572 fps was added to the velocity in the direction defined by $\Delta\theta_2 = 42.7^{\circ}$ and $\Delta\psi_2 = -125^{\circ}$. The resulting transfer orbit had an inclination of $118.4^{\circ}$, more than the required $116.6^{\circ}$ and, furthermore, the apogee altitude was 75,000 n mi. Note that for this case the transfer orbit apogee inequality constraint (7) is satisfied as an equality constraint, that is, $c_6(x)$ is in the basic set of constraints $B^*$. The third burn was again a retro maneuver, adding 3522 fps, in a direction yawed $187^{\circ}$ from the inertial velocity, and pitched down $21.4^{\circ}$.

Figure 4 presents a comparison of the minimum total velocity increments for two and three burn transfers to a range of final orbit inclinations. Notice that there is a reduction of approximately 5.12% in the total $\Delta V$ required to perform the mission when the three burn approach is used for a $26^{\circ}$ plane change ($i = 63.4^{\circ}$). The percentage improvement increases dramatically when the plane change is $79.2^{\circ}$ ($i = 116.6^{\circ}$) to 37.5%. It should be noted that the 5.12% reduction in total $\Delta V$ results in nearly 20% more payload capability for typical IUS configurations.

It is interesting to observe that for a plane change greater than $41^{\circ}$, the transfer orbit apogee altitude constraint was in the optimal basis, whereas for a plane change less than $41^{\circ}$ the constraint was not violated.

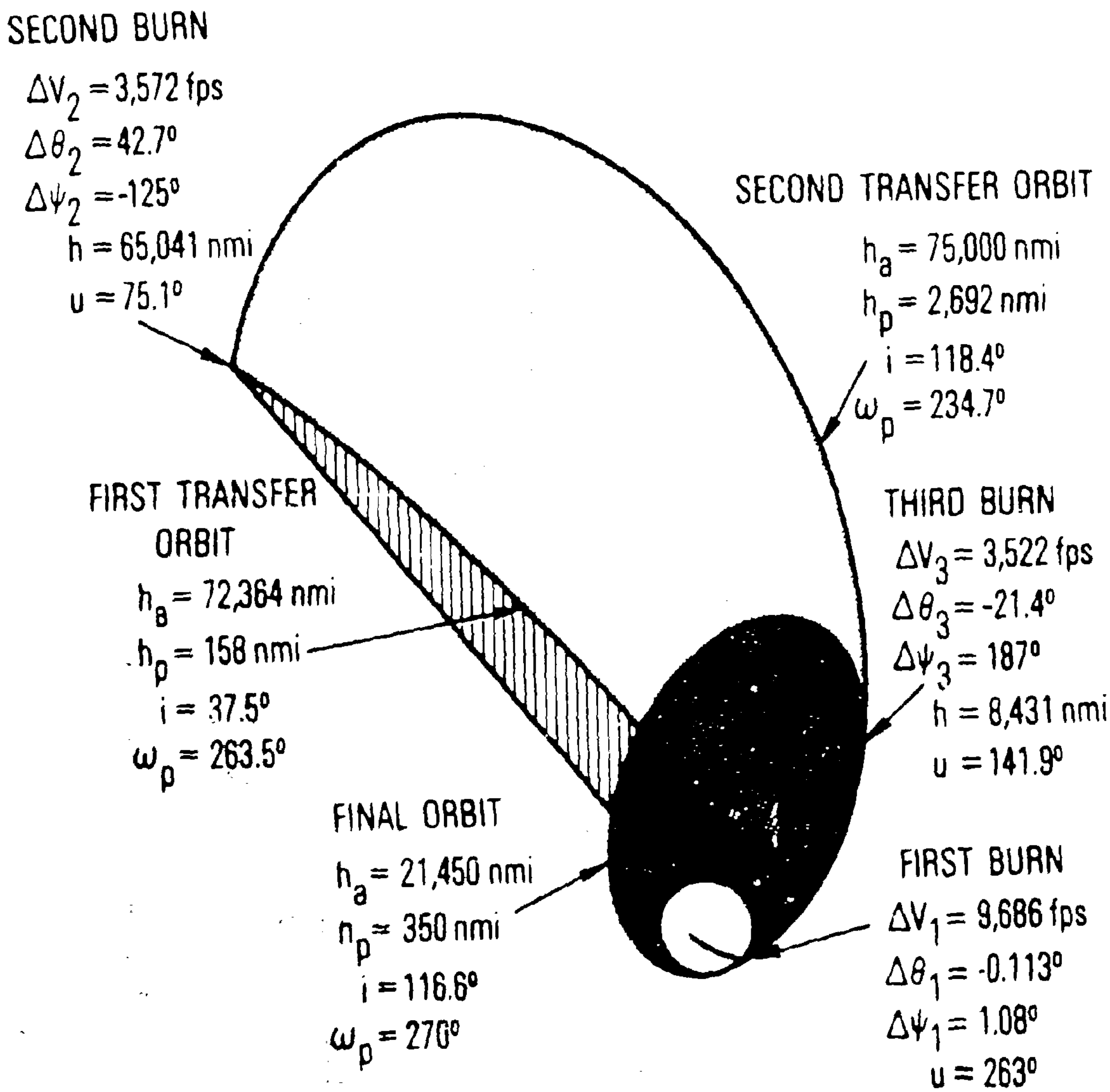


**Figure 3.** Optimal Three Burn Transfer, $i = 116.6°$

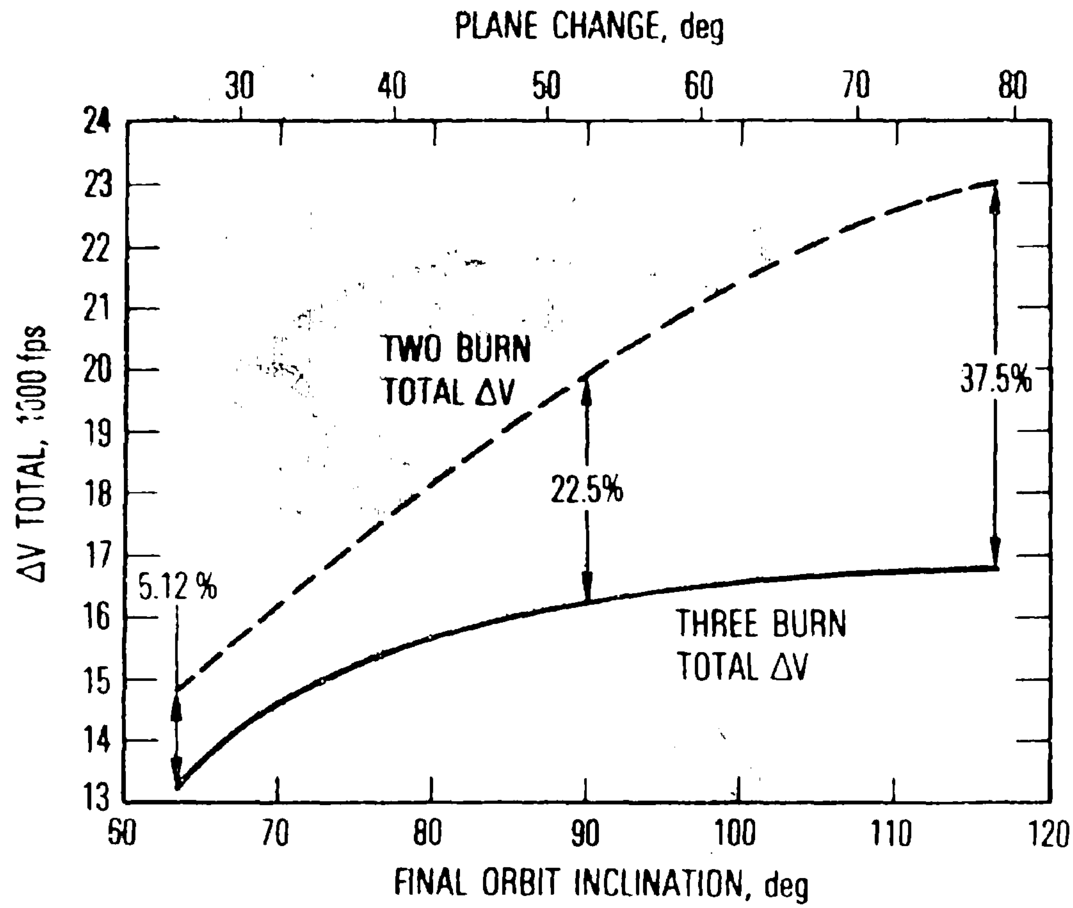


Figure 4. Impulsive Burn Incremental Velocities

The numerical sensitivity of the solution in this region is apparent from the rapidly changing behavior of the variables in this region. It seems clear that if the constraints did not prevent escape, the optimal solution would be to perform the second burn at an infinite altitude. In this situation one could accomplish all of the plane change with $\Delta V_2 = 0$. This of course is not a realistic solution to the problem. In general, for a three burn transfer requiring a plane change, one can expect that for a large enough plane change the optimal solution requires the second burn at infinity. The threshold value for the specific orbit transfer considered was approximately $41^{\circ}$ plane change.

## 5. EXTENSIONS AND GENERALIZATIONS

### a) Multiple Stage Vehicles

The results presented so far do not involve the vehicle mass in any of the computations. When dealing with a restartable single stage vehicle, this approach can be valuable. However, when a multiple stage vehicle is considered, the mass changes that occur at staging cannot be overlooked. In particular one can consider computing the $\Delta V_j$ for an N-stage vehicle according to the equations.

$$\Delta V_j = g_o I_j \ln \left[ \frac{J + \sum_{k=j}^{N} (P_k + S_k)}{J + \sum_{k=j}^{N} (P_k + S_k) - P_j} \right], \quad j = 1, 2, \ldots N, \tag{17}$$

where J is the weight of the payload, $P_k$ the weight of the propellant in the k th stage, $S_k$ the weight of the structure of the k th stage, $I_k$ the effective specific impulse of the k th stage and $g_o$ the gravitational constant.

By using the equations (17) to define the velocity increments in the trajectory simulation described in Section 2a) one can consider a number of different optimization problems. Specifically, one can pose a maximum payload orbit transfer by defining the objective function in Eq. (1) to be the payload J. Since the velocity increments are defined by Eq. (17), they are not treated as optimization variables. Instead the

variable set must contain the payload J. When designing a vehicle one may also want the propellant and/or structure weights $P_k$ and $S_k$ to be treated as variables. Cases have been run using each of the different sets of variables, for preliminary design analysis of the IUS vehicle. It should be clear that the maximum payload capability to a given orbit does not necessarily result by flying a minimum $\Delta V$ orbit transfer, unless the vehicle's weights $P_k$ and $S_k$ have been specifically designed for that orbit.

b) Finite Burn and Oblate Earth Effects

In the simulation an impulsive velocity approximation to a finite burn was used, and oblate earth effects were neglected in the geopotential model. To assess the effect of these approximations, the results were compared with those obtained from a detailed simulation. The variables for the detailed simulation included pitch and yaw attitudes at the beginning of each burn, constant pitch and yaw rates during the burns, the burn times, and location of each burn. The payload difference for typical missions was quite small (normally within a few percent).

c) Mirror Image Solutions

It should be observed that the optimization algorithm used to obtain the results presented only obtains a local optimum point. Another local solution to the problem does exist which we refer to as a mirror image solution. Because of the problem symmetry, the optimal value of the objective function is the same, although the location of the burns are different. For example, in the mirror image solution to the results presented in Fig. 2, the argument of latitude at the start of the second burn is in the second quadrant rather than in the first. Similarly, the argument of latitude for the beginning of the third burn is in the first quadrant rather than the second. Although the objective function is the same between the mirror image and the regular solution, there may be some other criteria for preferring one to the other. For example, the total transfer time or ground track for one case may be more desirable than the other.

## 6. SUMMARY AND CONCLUSIONS

The report outlines a nonlinear programming approach for obtaining the optimal three burn solutions to a class of orbit transfer problems requiring large changes in orbital inclination. For this type of problem, the first burn raises the apogee altitude of the first transfer orbit, and the second burn is performed at a high altitude. Since the velocity is low, most of the plane change can be efficiently accomplished by this burn. The final burn thus decreases the energy to meet the final orbit constraints. There can be a significant $\Delta V$ benefit derived from a three burn transfer with respect to a two burn transfer as Figure 4 indicates. Generalization of the approach to multiple stage vehicles as as well as more detailed trajectory simulations is discussed. The method is currently being used to solve a wide variety of trajectory optimization problems of varying complexity.

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